THE PRINCIPLE OF THE CONSERVATION OF ENERGY.

FROM THE POINT OF VIEW OF MACH'S PHENOMENO-LOGICAL CONCEPTION OF NATURE.

THE need of an epistemological investigation of the domain of the exact sciences, which has recently been making itself very vividly felt, like similar aspirations of universal and wide-reaching import, has found expression in many varied forms. Leaving out of account the original fundamental ideas of the great inquirers, which afford at all points *aperçus* of epistemological inquiries, the real era of the development in question began with Faraday, Lord Kelvin and Maxwell—although it was not consciously pursued until the present day, when a number of prominent inquirers began to investigate epistemologically the foundations of the exact sciences, not as a matter of supererogation, but as a definite end, sufficient in itself.

In two branches of knowledge which stand in intimate relation with physics, this work had been attempted at a much earlier date; namely, in philosophy and mathematics. In the first field, Berkeley, Hume and Kant had subjected the theory of knowledge to thoroughgoing scrutiny and criticism, and in so doing had at least demonstrated the necessity of such investigations. A critical examination of the foundations of mathematics by Abel, and by Weierstrass and his school, had proved amply that successful progress in the domain of mathematics was by no means a conclusive demonstration of the solidity of its foundations. It follows at once from the results of these inquiries that a critical examination of the epistemological structure of physics likewise is an imperative necessity, and has in

no sense been rendered redundant by the steady progress made in this science.

MACH'S EPISTEMOLOGICAL WORK.

Ernest Mach was undoubtedly the first inquirer to discern clearly the necessity of a reform of the current conceptions of the fundamental principles of physics and to make the epistemological investigation of these notions an independent object of inquiry. For forty years now, in the prosecution of this task, he has produced an imposing array of works and memoirs. For a long time he remained isolated, and, as he himself tells us, for many years the only reception his ideas met with from his colleagues was a shrug of the shoulders. But gradually the number of those who either partly or wholly agreed with his views increased; nay! he was ultimately successful, even, in discovering kindred ideas among inquirers of an earlier period; principally, B. Stallo, a German-American whose *Concepts and Theories of Modern Physics* appeared in 1881 and who in very many points, at times even in minute details, is in surprising accord with Mach; and the celebrated English mathematician Clifford (died 1879 in Madeira), who took a related, if not an identical, point of view with that of Mach.[1]

Mach found substantial support for his views also in the philosophical school of Avenarius, while more recent thinkers, like H. Cornelius in Munich, exhibit even greater affinities. In England, again, Karl Pearson has expressed his full agreement with Mach's views on the epistemological foundations of physical science in his book *The Grammar of Science,* the first edition of which appeared in 1892, and the second enlarged edition in 1900.

POSTULATES, HYPOTHESES, AXIOMS AND NATURAL LAWS.

The discussions on this subject have of late centered chiefly about the definitions of such conceptions as "axioms," "postulates," "hypotheses" and "natural laws," the endeavor being to formulate

---

[1]Goethe and Julius Robert Mayer had given expression to views of a similar character. So also had Adam Smith.

precisely the functions which these traditional conceptions of physics fulfill, and to what extent they are permissible and serviceable.

It is of importance to understand in the first place that the notion of "axioms" is an entirely superfluous one. Grassmann demonstrated this assertion with regard to the formal sciences as early as 1844 in his *Ausdehnungslehre,* a book which is collaterally of great philosophical importance. At the head of arithmetic, he argues, may be placed, instead of the customary *axioms,* the following two simple *definitions:*

1. Any two numbers are said to be equal numbers, when, in a given calculation, one of them can be substituted for the other.

2. A quantity is said to be greater than another quantity when the latter is representable as a part of the former.

From these definitions may be deduced logically the principle (frequently cited as an axiom in the same connection), that "like changes performed on like quantities give like results."

Similarly the axioms of logic may be expressed in the form of definitions. The principle of identity is correctly expressed by asserting that "a is always *a*," and furnishes a definition of the concept of substance; the principle of contradiction is, according to the varying form which we give to it, either equivalent with the first principle or it contains a definition of negation.

Now, coming to physics, the highest and most general principles of this science likewise contain definitions, but their contents are not fully exhausted by these definitions; they are not pure definitions, but are characterized by the so-called principle of "particular determination" propounded by L. Lange and accepted by Mach.[1]

Thus, for example, before the law of inertia can be made to have a meaning we are in need of a system of co-ordinates and a scale of time. For establishing this principle we are in need of several bodies, which the law of inertia, as a result of the definitions, must hold valid.

This is most easily seen in the case of time. The concept "uniform" must first be defined. For this purpose may be chosen a body,

---

[1]See *The Science of Mechanics,* Chicago, 1902, 2d edition, page 544.

moving with respect to a given system of co-ordinates, the displacements of which body are considered as the measures of time. That motion, then, of a second body is called "uniform" of which the positional displacements are proportional to the displacements of the body of reference. Such a body of reference, for example, is the rotating earth, and the motion of a body is said to be "uniform" when the displacements which it undergoes between the same points of it are proportional to the angle of hours reckoned from the vernal equinox. Absolutely, there is no assignable meaning in speaking of a uniform motion *per se*. The law of inertia has, consequently, with reference to any second new body, the significance of a natural law. It makes with respect to that body an assertion which can be demonstrated experimentally.

Recapitulating, then, we may say that the law of inertia, and, analogously also, the other so-called "axioms" or "postulates," have only a *partial* definitional value, in that, first, they define certain conceptions, but in that, secondly, their contents are not entirely exhausted with the formulation of these definitions; nay, that, on the contrary, inasmuch as the law asserts the universal applicability of these concepts, it employs assertions, of which the actuality may be controlled by experiment. Principles of this character have, therefore, a validity which stands in part only the test of experience (that is, they may not always be borne out by experience), which isolated exceptions it may be possible to harmonize with the principles by the subsequent construction of new and other principles of a more special character. The only genuine test of the value of principles of this sort is the possibility or impossibility of deducing from them a system, or, in better phraseology, of erecting a system on the foundations which they furnish.

Consequently, the decision with regard to the value of the highest and most general principles of physics rests primarily with the outcome of repeatedly continued experimental tests which directly confirm or refute the truth of the more specific principles. The general principles are demonstrated to be serviceable implements of science when the special principles confirmed by experience permit of their being arranged, along with the former, into a single system

in which the so-called "postulates" possess the most comprehensive and most general significance.

From this state of matters the conclusion follows immediately that the difference between "postulates" and "natural laws" is a graduated one only. For, that the latter likewise possesses axiomatic, or, as we might more correctly say, definitional, significance, is upon the face of it obvious, though the statement may be enhanced in lucidity by the consideration of its special application to Coulomb's law. Writing this law in the form

$$f = \frac{M_1 M_2}{r^2}$$

where f represents the magnitude of the force M_1 M_2 the numbers measuring the masses, and r the number measuring the distance, by which assertion (on the mere assumption that it is possible to produce equal masses) the unit of mass, but not multiples of that unit, are defined, it will be apparent that it is impossible to determine the numbers which are the measures of these masses by the consideration of two masses only, for the reason that a product can be separated into two factors in different ways. Three masses may be combined in three ways, and hence permit of being uniquely determined from the three equations so obtained; in other words, the law is just sufficient for the definition of the masses of three bodies. The experimental verification of the proportionality of force to product of masses is still impossible so long as we have three bodies only; for these bodies the existence of the law is given by the definition of the three masses. Not until four masses are presented does the law afford an assertion that can be experimentally verified; for these four bodies may be combined in six ways, giving six equations, of which four are employed for determining the unknown quantities M_1, M_2, M_3, M_4, and of which the other two may be employed in the verification of the law by experience.

THE PRINCIPLE OF THE CONSERVATION OF ENERGY.

Many and numerous as are the applications which the principle of the conservation of energy has found in all provinces of

physics; great as its import for science in general has become, nevertheless the conceptions of physicists regarding the theoretical nature of this principle are so far apart and discordant that, as Mach says, "opinions regarding the foundations of the law of energy still diverge greatly even at this late day." Similarly variant are the views regarding the position which this principle occupies in theoretical physics. One school of physicists regards it as the highest and the exclusive law of physics, ascribing to it an almost axiomatic import, whilst other inquirers—to mention only Mach, Hertz and Boltzman—have believed themselves called upon to take a most decisive stand against exaggerations of the scope of this principle. Indeed, so far has the conceptual investigation of this principle been neglected that to-day even the form of enunciation in which it appears in most text-books can lay little claim to precision, let alone to correctness. Considered rigorously and unprejudicedly, therefore, the principle of the conservation of energy is wrongly formulated; at least, to express it in Mach's words, "there are limits beyond which the principle can be only artificially maintained."

Nevertheless, it is difficult to define these limits with anything like precision. The fact is that the principle is in contradiction with the second fundamental law of the mechanical theory of heat, and this contradiction must be removed if the logical harshness of the formulation of the principle is to be removed. But the source of the contradiction in question can be found nowhere else than in the concept of energy itself, from which all our assertions are predicated. In point of fact the elucidations of the first inquirers in this field, for instance, the works of Clausius, do not contain this contradiction; but, then, the term "energy" does not as yet appear in them.

What now is "energy?" It is certainly not a substance; certainly not a substantial or real property inherent in a body, as is erroneously believed in many quarters. It is a concept formed, like all other physical concepts, on the basis of definite given facts, though not ceasing on this ground to be an arbitrary creation of our intellect. And in point of reality the concept of "energy" has been so formed and selected that it shall supply fully the needs of the principle of "energy."

The facts upon which its formation rests are the following: In the domain of mechanics exists a large number of motions of which the property of reversibility is a characteristic feature, just as in the cycle of Carnot. When, for example, a body falls a certain distance, it can be made to rise again the same distance by simply reversing the direction of its velocity. This peculiarity of the phenomenon of motion was conceived as a *capacity* of the body, and was designated "its living force." Inasmuch as the body has at every point of its path "the power" to evoke another like motion, differing in direction only, inquirers were led to speak of the constancy of living force, always positive because direction was not regarded. Hence resulted the principle of the "conservation of living forces," the applicability of which is strictly limited to reversible processes of the kind described.

In the case of friction, which is a non-reversible process, the principle does not hold—a conclusion which is mathematically expressed by saying that the principle loses its validity as soon as forces appear which are dependent on velocity. But since, in such cases, a body loses living force, it was an obnoxious suggestion to regard the heat which made its appearance as the *equivalent* of that force.

The word "equivalent" must not be misunderstood. Heat is not a real "equivalent," for the reason that it is impossible to reverse the process, and because also the process can take place only in one direction. Nevertheless, it is possible to conceive this heat as a partial summand of the total energy of the body and to add it to energy of some other kind. Again, it must not be forgotten that this heat, or this part of the energy of the body, represents no capacity whatever to perform work, but has simply the significance that it can, and actually does, become the equivalent of work when a transformation of heat into work takes place. This condition can under no circumstance be left out of consideration (as has hitherto been the custom in defining energy), for the reason that there is absolutely no necessity that the said transformation should be possible. Consequently, the principle of the conservation of energy simply informs us, when a transformation of energy takes place, what the ratio is in which that transformation is accomplished.

It may be remarked further, that the truth of the principle of energy assumes that the manner of measuring homologous quantities is the same in the different departments of physics. As a matter of fact, the modes of measuring quantities of heat and electrical work are homologous with the modes of measuring mechanical work. But this would not be the case were not the potential, or the difference of level, of the electrical states of two bodies measured by units of mechanical work—for which state of things there is no real logical necessity. Otherwise the law of energy would not be applicable to electrical phenomenon, and, in place of simple proportionality, which can be transformed into equality by the simple choice of an appropriate unit of measure, would be substituted a more complicated function.

The stability of the law of energy is thus essentially dependent on our arbitrarily selected definitions of the fundamental concepts of physics; the possibility of its existence is dependent on the presence of an equation between the different groups of physical concepts of measurement.

A genuine law of nature, in the sense usually supposed, this principle is not, even though certain actual facts are at the basis of the assumption of its validity.

Ordinarily we think, when we hear the law of the conservation of energy—not so much of energy in the sense above described as of energy in the sense of *capacity to perform work*—which, as we have seen from what has gone before, are two distinctly different things. But if in the place of energy we take as the essence of our principle capacity to do work, then this principle assumes an entirely different form. In this last case nothing more can be generally affirmed than that the capacity to do work has remained constant, when the passage from the first to the second state as well as from the second to the first is *possible*, or, when, as we say, we are concerned with a reversible cycle.

The principle of the constancy of capacity to do work *is thus restricted to cyclic processes,* and is none other than Clausius' principle of the equivalence of transformation, which is one of the forms of the second law of the mechanical theory of heat.

Processes which are not reversible, as, for example, friction, or the passage of heat from higher to lower temperatures, result in the *dissipation* of capacity to do work. Since, in considering the principle of energy, it has been customary to adhere rigidly to this last-mentioned meaning of the word, it will be apparent that the principle of the conservation of energy, as thus formulated, was incorrect, and we now know also what the limits mentioned by Mach are, "beyond which this principle lost its validity."

It will at the same time be apparent that this second principle is a more appropriate expression of a characteristic peculiarity of our nature than is the second—which furnishes us merely with an equation between our own concepts. The reason for this is that the second principle is more intimately associated with the phenomena of nature and is not so much concerned with the properties of bodies as they are assumed to be by us.

The results of the foregoing inquiry are, then, the following:

1. The principle of the conservation of energy is in its present form incorrect.

2. A distinction must be made between "energy" and "capacity to do work."

3. Whether the first or the second concept is embodied in the principle mentioned, are obtained in its place two laws; namely, the first and the second laws of the mechanical theory of heat.

Furthermore, and finally, the special importance of reversible cyclic processes in all the departments of physics is made apparent. Thus, without a consideration of the concept of reversible processes, it would appear to be utterly unfeasible to attempt a formulation of the principle of energy, and on the grounds which have been adduced the introduction of the two first laws of the theory of heat into instruction in physics in secondary schools would appear to present no objections.

HANS KLEINPETER.

GMUNDEN, AUSTRIA.